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APPLICABILITY OF BOUNDARY-LAYER THEORY TO CALCULATION OF HEAT TRANSFER UNDER SEPARATION CONDITIONS

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The conditions are examined under which methods and relations developed for attachment flow are applicable to regions of separation flow.

Recently, the number of experimental and theoretical studies has been increasing where the authors examine the possibility of calculating the processes during separation by the method based on the theory of a boundary layer in non-separation streamlining, this method having been rather thoroughly tested for a wide range of applications. The problem has not only great practical but also fundamental theoretical importance, inasmuch as studied pertaining to it will reveal differences between the physics of separation flow and the physics of attachment flow. No single view on this matter has yet been developed, apparently because of the multitude of modes and forms of separation flow. There is no doubt that the possibility of applying the relations varied for attachment flow to conditions of separation flow must be examined individually in each specific case.

An important aspect of the problem is determining whether there exists an analogy between friction and heat transfer in separation flow. For determining the friction in this study the authors used a known indirect method [1, 2]. In accordance with that method, the frictional velocity (or dynamic velocity) was selected so as to ensure the required direction of the measured velocity near the wall. The method was used here for determining the friction in two-dimensional grooves streamlined by a compressible gas. The study covered a wide range of parameter variation: of the relative groove depth $\bar{H} = H/L$ from 0 to 1.0 and the Reynolds number $N_{Re} = u_e x/\nu$ in the stream core from $2.5 \cdot 10^5$ to $3.5 \cdot 10^6$, with the Mach number N_{Ma} equal to 3.5, 4.0, and 4.5 successively.

On the graph in Fig. 1 are indicated the readings of velocity obtained with a total-head Pitot tube near the wall at four sections along the x-coordinate in grooves of various depths. The frictional velocity had been selected so as to make the experimental points fit on the Karman curve for a buffer layer with $u^* = -3.05 + 5$ ($\ln y^*$) within the $5 \leqslant y^* \leqslant 30$ range, with $u^* = u/u_\tau$ and $y^* = yu_\tau/N$.

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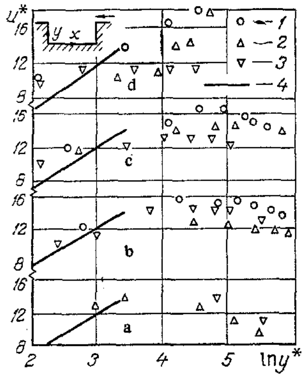

Fig. 1. Determination of frictional velocity from readings of velocity at bottom of groove: 1) $\bar{H}$ = 1.0, 2) $\bar{H}$ = 0.486, 3) $\bar{H}$ = 0.104; 4) Karman curve for buffer layer $u^* = -3.05 + 5(\ln y^*)$; a) x/L = 0.167; b) x/L = 0.389; c) x/L = 0.611; d) 0.833.

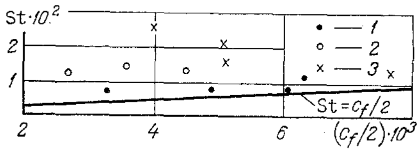

Fig. 2. Relation between heat transfer and friction: 1) $\bar{H}$ = 1.0; 2) 0.486; 3) 0.104.

It was possible to approximate the Karman curve for a buffer layer most closely in the intermediate sections at x/L = 0.389 and x/L = 0.611 at the bottom of grooves. The approximateness of the method used here is obvious, but the validity of its application here is confirmed by a comparison of the magnitude of shear stress according to calculations in this study with that according to measurements by the electrodiffusion method in another study [3]. There the authors measured the friction coefficients at the walls of a square and deeper grooves. A comparison of our data in the form of the $2\tau_u Re_L^{0.23}/\rho u_e^2 = f(x/H)$ relation with the data in that other study [3] reveals a close qualitative agreement, which including the Prandtl number N_{Pr} ($c_f/2 \approx N_{Pr}^{-0.67}$) will further confirm also a satisfactory quantitative agreement.

Earlier reports [4, 5] cover the results of an experimental study of the heat transfer in grooves over the same range of $\bar{H}$, N_{Re}, and N_{Ma} as in this one. In order to facilitate an analysis of the relation between friction and heat transfer in a groove, the results of these experiments with heat transfer and the results of friction measurements shown here have been presented in the form of the $N_{St} = f(c_f/2)$ relation (Fig. 2). The relation shown here graphically and describable by the equation of a straight line ($N_{St} = c_f/2$) corresponds to the Reynolds analogy. All experimental points lie above this straight line, which suggests that the Reynolds analogy does not apply to such a type of separation flow and application of this analogy here will result in underestimates of the heat transfer intensity.

The analogy between friction and heat transfer is most closely approximated by points characterizing the relation between these two processes in a square groove. This indicates that it may be possible to approximately calculate the heat transfer in deep grooves ($\bar{H}$ = 1.0) on the basis of available data on the distribution surface friction in them. In shallower grooves there appear appreciable deviations from that analogy and the latter can be used only with corrections, the magnitude of these corrections decreasing as the depth of a groove increases. Therefore, the applicability of the Reynolds analogy must be examined individually in each specific case. It is necessary to identify the factors (determine their magnitudes and distributions) which may cause departures from that analogy and then account for them in corrections. Outstanding among these factors are pressure gradient [6], nonisothermality (T_w/T_e), and the Prandtl number [7]. Calculation of such corrections according to certain known methods [6, 7] (on the assumption that the effect of the disturbing factors on the analogy is the same in separation flow and in attachment flow) has revealed that their magnitude depends on x/L, varying from 1.05 to 1.27 for $\bar{H}$ = 0.486 and from 1.05 to 1.65 for $\bar{H}$ = 0.104, respectively. However, the deviation of experimental point from the Reynolds analogy (Fig. 2) exceeds these corrections. One can conclude, therefore, that separation flow has peculiar features which cause departure from the analogy or that the effects of said factors ($\partial p/\partial x$, T_w/T_e, N_{Pr}) on the relation between heat transfer and friction are different in separation flow and in attachment flow.

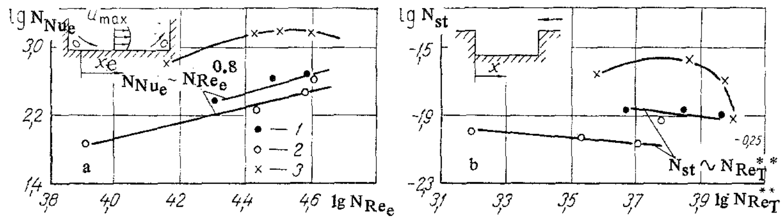

Fig. 3. Use of integral methods for calculation of heat transfer in grooves: a) method of effective length; b) method of local modeling; 1) $\bar{H}$ = 1.0; 2) 0.486; 3) 0.104.

Quite interesting is the question as to whether the similarity equations for plates are applicable to separation flow. On account of the strong irregularity of parameters in a groove, it is apparently worthwhile to isolate structurally homogeneous segments where there are no large pressure and velocity gradients and where the direction of flow does not change as some analog of a plate in terms of flow pattern. Such segments include, for instance, the region of a boundary layer formed at the bottom of a groove through interaction of the bottom surface and the primary vortex.

In two other studies [8, 9] there has been outlined a method of calculating laminar and turbulent boundary layers in streams with pressure gradient. The gist of this method is to apply the relations for a straight plate at each point of the given body, provided that the true length along the surface has been replaced with some effective length based on integral relations. In the simple case of two-dimensional flow this effective length is related to the stream parameters through the equality

$$x_e = \frac{1}{up} \int_0^x updx. \tag{1}$$

The effective length for reverse flow in a groove was read from the starting point of the boundary layer, this point having been determined from the results of visualization by means of flash photography and according to the oil film method (Fig. 3a). Processing of the experimental data according to this method has yielded a qualitative relation between the Reynolds number and the Nusselt number in satisfactory agreement with the law of nonseparation-flow streamlining of a surface by a turbulent boundary layer ($N_{Nu} \sim N_{Re}^{0.8}$) (Fig. 3a). The evident stratification of points at various depths of a groove suggests that in an analysis of separation-flow streamlining of a groove there must be taken into account additional factors influencing the heat transfer such as, for instance, a higher turbulence level. Unfortunately, the scarcity of tests performed with measurement of local velocities in grooves makes it impossible to identify the effect of these factors quantitatively and to establish a generalized relation including parameters of the boundary layer at various depths. Nevertheless the conclusion about applicability of the relations for a plate to streamlining of a groove, based on analysis of data (Fig. 3a), is entirely legitimate. It is furthermore noteworthy that this method, one basic premise here being the absence of any secondary flow (hypersonic streamlining of bodies oriented at wide angles of attack, of bodies with very blunt surfaces, etc.), applies only to regions with one particular mode of flow. Thus the linear relation between log N_{Nu_e} and log N_{Re_e} corresponding to the relation $N_{Nu_e} \sim N_{Re_e}^{0.8}$ holds true within the zone of the primary vortex in a square groove. At the depth $\bar{H}$ = 0.486 here the last point, which lies within the zone of the secondary vortex, departs from the general pattern. This departure is even farther at $\bar{H}$ = 0.104, inasmuch as the dimensions of the secondary vortex become comparable with those of the primary one.

Therefore, application of the method in studies [8, 9] to grooves must be based on a differentiated approach to the various zones of the boundary layer, taking into account the direction of the stream within each zone.

In the course of this study was also checked the possibility of calculating the heat transfer in a groove on the basis of the theory of local modeling for a plate. The integral relation for energy in this case is expressed as [10]

$$\frac{dN_{Re_T^{**}}}{dx} + \frac{N_{Re_T^{**}}}{\Delta T} \frac{d(\Delta T)}{dx} = N_{Re_L} N_{St}, \tag{2}$$

where $N_{Re_T^{**}} = \frac{\Delta T^{-1}}{\lambda N_{Pr}} \int_0^x q_w(x)\,dx$ and ΔT is the temperature drop from the wall to the surrounding stream.

It may be noted that only in the case of sufficiently deep grooves ($\bar{H}$ = 0.486, 1.0) does the Stanton number depend on the Reynolds number to a degree approaching the relation $N_{St} \sim (N_{Re_T^{**}})^{-0.25}$ for streamlining of a plate. In a shallow groove ($\bar{H}$ = 0.104), just as in Fig. 3a, the relation departs from that for streamlining of a plate.

NOTATION

H, depth of a groove; L, length of a groove; u_e, velocity of the stream core before separation; ν, viscosity of the fluid; λ, thermal conductivity of the fluid; x, longitudinal coordinate on the bottom of a groove measured from the front wall; u is the local velocity; $u_\tau = \sqrt{|\tau_w|/\rho}$, frictional velocity; τ, shear stress; ρ, density of the fluid; y, normal coordinate; N_{St}, local Stanton number; c_f, frictional drag coefficient; p, pressure; N_{Nu}, Nusselt number; N_{Re}, Reynolds number; $N_{Re_T^{**}}$, Reynolds number based on the energy-loss layer thickness; q(x), local thermal flux; $\bar{x} = x/L$, referred longitudinal coordinate; and subscripts e, w refer to stream core and wall, respectively.

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